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Solving of distribution networks reliability used multi-criteria analysis

Streszczenie: *(**Wyznaczanie niezawodności sieci rozdzielczych za pomocą metody analizy wielokryterialnej**). Zwiększenie niezawodności sieci elektroenergetycznych, nie tylko średnich napięć, można osiągnąć na przykład przez zastosowanie nowych urządzeń wyposażonych w zdalnie prowadzony nadzór. Taka modernizacja jest jednak kosztowna. Dlatego przywiązuje się dużą wagę do wyboru nowych urządzeń i ich lokalizacji. Do tego celu przydatna jest metoda analizy wielokryterialnej.*

Abstract*. The increase of reliability of electrical networks, not only medium voltage, can be achieved for example by use of new components, above all remote-controlled elements. That modernization is considerably expensive. Therefore close attention is paid to selection of new components and place of their position. The use of methods of multi-criteria analysis is suitable for this decision making.*

Słowa kluczowe: niezawodność, urządzenia zdalnie kontrolowane, analiza wielokryterialna
Keywords: reliability, remote-controlled elements, multi-criteria analysis.

Introduction

The multi-criteria analysis (MCA), as the name itself indicates, deals with the evaluation of particular alternatives according to several criteria. The term "alternative" designates each of the solutions of the selection report. The "criterion" is a property that is being evaluated with the given alternative. To each criterion such as weight is assigned that expresses the importance of particular criteria with regard to the others.

Following methods appear as advisable methods for solution of existing problems:

- WSA method – Weighted Sum Approach
- IPA method – Ideal Points Analysis
- TOPSIS method – Technique for Order Preference by Similarity to Ideal Solution
- CDA method – Concordance-Discordance Analysis

Multi-criteria analysis [1]

The initial step of each MCA analysis is to form an evaluating matrix - the elements of it reflect the evaluation of particular criteria for each alternative. The matrix S consists then of elements S_{ij} where i = 1, .. I alternatives and j = 1, .. J criteria.

The evaluating matrix:

$$S = \begin{matrix} S_{11} & \dots & S_{1J} \\ \dots & & \\ S_{I1} & \dots & S_{IJ} \end{matrix} \qquad (1)$$

Because particular evaluations are not mostly measured against the same units, it is necessary to carry out the standardization of the matrix to the standard condition. For the case when the higher evaluation of the criterion means also the better evaluation (i.e. 1 = max, 0 = min), we can write the standardization as follows:

$$e_{ij} = \frac{S_{ij} - \min_i S_{ij}}{\max_i S_{ij} - \min_i S_{ij}} \qquad (2)$$

In the contrary case, when the higher evaluation means the worse evaluation (i.e. 1 = min, 0 = max), the standardization will be as follows:

$$e_{ij} = \frac{\max_i S_{ij} - S_{ij}}{\max_i S_{ij} - \min_i S_{ij}} \qquad (3)$$

Ideal Point Analysis (IPA) [1]

The Ideal Point Analysis rests upon the deviation between the set of ideal solutions and the set of effective solutions. Although the ideal solution surely almost does not exist, it serves as an important reference model. The best compromise solution is determined as that solution that is distant least from the ideal one. The increasing distance from the ideal solution for factors located upper on the scale of importance induces greater consequences than the increasing distance from the ideal solution for factors located lower on the scale of importance. The IPA model can be described as follows:

$$d_i = \sum_{j=1}^{J} v_j \cdot (1 - e_{ij}) \qquad (4)$$

where: d_i - the minimum distance from the ideal solution
v_j - the j-th weight of criterion
e_{ij} - the standardized evaluation

We rank the alternatives according to the growing distance from ideal solution.

Weighted Sum Approach (WSA) [3]

The Weighted Sum Approach method comes from principle of maximization of benefit, simplification of this method is that it is assumed only linear function of benefit. Process of this method is comfortable to IPA method; resulting sequence of alternatives is opposite. Benefit of variant is following:

$$u_i = \sum_{j=1}^{J} v_j \cdot e_{ij} \qquad (5)$$

We rank the alternatives according to the declining value of benefit.

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [3]

In case of TOPSIS method there's the question of principle of maximization of distance from ideal variant again. The ideal variant means that all criteria have the best assessments. Ideal variant is mostly suppositional; the best of variants is that one which is the least distant from ideal variant. Vector ($H_1, H_2, \ldots, H_J$) represents ideal variant, vector ($D_1, D_2, \ldots, D_J$) represents basal variant.

The initial step is the construction of criteria-normalised matrix $\boldsymbol{R}=(r_{ij})$, following formula is proposed for calculation of normalised values:

$$r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{I}(y_{ij})^2}} \quad (6)$$

After this transformation vectors with unit size are in columns of matrix R.

The next step is calculation of criteria-weighted matrix W so that each j-th column of criteria-normalised matrix R multiplies by appropriate weight v_j.

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1J} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ w_{I1} & w_{I2} & \cdots & w_{IJ} \end{bmatrix} = \begin{bmatrix} v_1 r_{11} & v_2 r_{12} & \cdots & v_J r_{1J} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ v_1 r_{I1} & v_2 r_{I2} & \cdots & v_J r_{IJ} \end{bmatrix} \quad (7)$$

Now we define the ideal variant ($H_1, H_2, \ldots, H_J$) and the basal variant ($D_1, D_2, \ldots, D_J$) respecting values of criteria-weighted matrix:

$$H_j = \max_i (w_{ij}) \quad (8)$$

$$D_j = \min_i (w_{ij}) \quad (9)$$

The next step is calculation of distance of variants from ideal variant:

$$d_i^+ = \sqrt{\sum_{j=1}^{J}\left(w_{ij} - H_j\right)^2} \quad (10)$$

and distance of variants from basal variant:

$$d_i^- = \sqrt{\sum_{j=1}^{J}\left(w_{ij} - D_j\right)^2} \quad (11)$$

The calculation of relative index of distance of variants from basal variant is the following:

$$c_i = \frac{d_i^+}{d_i^+ - d_i^-} \quad (12)$$

We rank the alternatives according to the declining indicator c_i.

Concordance-Discordance Analysis (CDA) [2]

The Concordance-Discordance Analysis is a method widely used in MCA. It consists of comparison of alternatives of pair selection. It measures the degree by which the alternatives of selection and the weights of factors prove or disprove the ratio between the alternatives. The differences in the weights of factors and in the evaluations of criteria are analysed by means of the procedures of concordance and discordance separately.

The index of concordance between the alternative A and the alternative B is defined as a proportion of the sum of weights of those criteria for which the evaluation A is greater than or equal to the evaluation B, and the sum of weights of all criteria.

The following is valid for the concordance index then:

$$C_{AB} = \frac{\sum v_j \ for \ e_{Aj} \geq e_{Bj}}{\sum v_j} \quad (13)$$

The index of discordance between the alternative A and the alternative B is defined as a proportion, where a numerator equals to the maximum difference between weighted evaluations, for which the evaluation A is less than the evaluation B, and the denominator is equal to the maximum difference between weighted evaluations of all alternatives for the criterion showing the maximum value of the numerator defined above. Thus we can write the discordance index as follows:

$$D_{AB} = \frac{D1}{D2} = \frac{max_j\,(v_j.e_{Bj} - v_j e_{Aj})\ for\ e_{Aj} < e_{Bj}}{max_i\, v_m.e_{im} - min_i\, v_m e_{im}} \quad (14)$$

where: $m = j$ at $D1$ = max.

The total concordance index of the alternative A can be obtained as a sum of all concordance indexes of the alternative A with regard to all the others:

$$C_A = \sum_{j=1}^{J} C_{Aj} \quad (15)$$

The total discordance index of the alternative A can be obtained as a sum of all discordance indexes of the alternative A with regard to all the others:

$$D_A = \sum_{j=1}^{J} D_{Aj} \quad (16)$$

Our goal is to rank the particular alternatives according to the maximum concordance index and the minimum discordance index. The resultant evaluation of the given alternative can be obtained in the following way:

$$CDA_i = I - C_i + D_i \quad (17)$$

We rank the alternatives according to the growing CDA value.

Weights of criteria [2, 3]

We can divide the methods of determination of weights as follows:

1) methods with nominal information about criteria
2) methods with ordinal information about criteria
3) methods with cardinal information about criteria
   a) sequence method
   b) spot method

c) methods of paired comparison
- method of Fuller triangle
- Saaty method
- method of geometric mean of rows

Methods of paired comparison are optimal for solution of mentioned problems.

Method of Fuller triangle

The principle – for each criterion j = 1, ..J the number of preferences of this criterion over the others is to be determined. The first step is the formation of a triangle that consists of all possible pairs of criteria. After that, of each pairs that criterion is designated that is considered to be more significant. For each criterion, we shall determine the number of preferences over the other criteria; this number will be designated n_j.

The total number of pairs under comparison is as follows:

$$N = \frac{J.(J-1)}{2} \tag{18}$$

The following relation defines the resultant weight v_j:

$$v_j = \frac{n_j}{N} \tag{19}$$

Saaty method

Principle: The calculation of weights consists in calculation of eigen vector, which comports with maximal eigen number of matrix of paired comparison A. The first step is creation of matrix of paired comparison A with elements A_{mn}, where m, n = 1, .. J. Element A_{mn} indicates the ratio of relevance of criterion m to criterion n, i.e. ratio of weight v_m to weight v_n. We select value A_{mn} from scale 1 to 9 when criterion m is major criterion n, in opposite case A_{mn} = 1/A_{nm}. The next steps are calculation of eigen vector and eigen number of matrix A:

$$A.\vec{x} = \lambda_{\max}.\vec{x} \tag{20}$$

We assign individual weights of criteria:

$$v_j = \frac{x_j}{|\vec{x}|} \tag{21}$$

where: j = 1, .. J, $|\vec{x}| = \sqrt{\sum x_j^2}$ - vector size

Method of geometric mean of rows

This is simple and unassuming method of determination of criteria weights from matrix of paired comparison A. It comes from calculation of geometric mean of each row of this matrix:

$$g_m = \sqrt[J]{\prod_{n=1}^{J} A_{mn}} \tag{22}$$

This is possible manner of standardization for accomplishment of condition $\sum_{j=1}^{J} v_j = 1$ for $v_j \geq 0$:

$$v_j = v_m = \frac{g_m}{\sum_{m=1}^{J} g_m} \tag{23}$$

where: j, m, n = 1, .. J

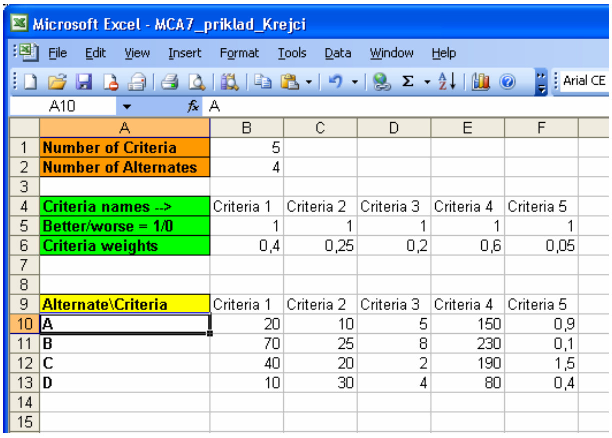
Description of MCA7 program

Program MCA7 was developed at Department of electrical power engineering. It allows calculation of mentioned methods of multi-criteria analysis:
- WSA method
- IPA method
- TOPSIS method
- CDA method,

and determination of weights of criteria by means of Fuller triangle method and Saaty method.

Input data

Input data is inserted into table of MS Excel program, as we can see in following Figure 1.



	A	B	C	D	E	F
1	Number of Criteria	5				
2	Number of Alternates	4				
3						
4	Criteria names -->	Criteria 1	Criteria 2	Criteria 3	Criteria 4	Criteria 5
5	Better/worse = 1/0	1	1	1	1	1
6	Criteria weights	0,4	0,25	0,2	0,6	0,05
7						
8						
9	Alternate\Criteria	Criteria 1	Criteria 2	Criteria 3	Criteria 4	Criteria 5
10	A	20	10	5	150	0,9
11	B	70	25	8	230	0,1
12	C	40	20	2	190	1,5
13	D	10	30	4	80	0,4
14						
15						

Fig. 1. Example of input data

Output data

Resulting sequence of alternatives from the best to the worst is positioned into table, as for example in Figure 2. Assumed and sequenced tables can be saved again as MS Excel document.

Weights of criteria

Weights of criteria can be calculated by both Fuller triangle and Saaty method, as it follows in next Figure 3.

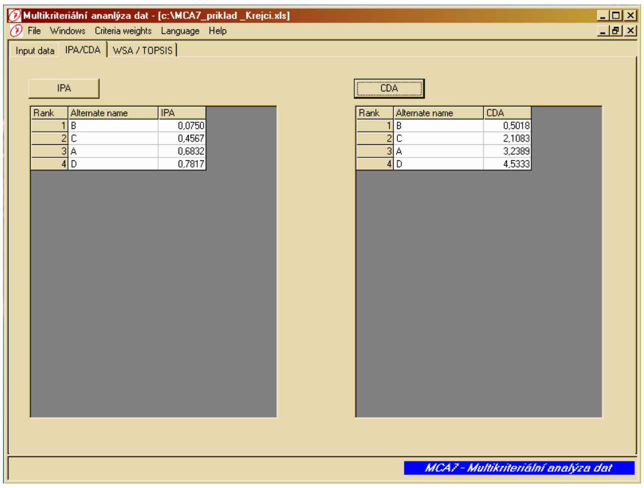


Fig. 2. Example of resulting sequence of alternatives

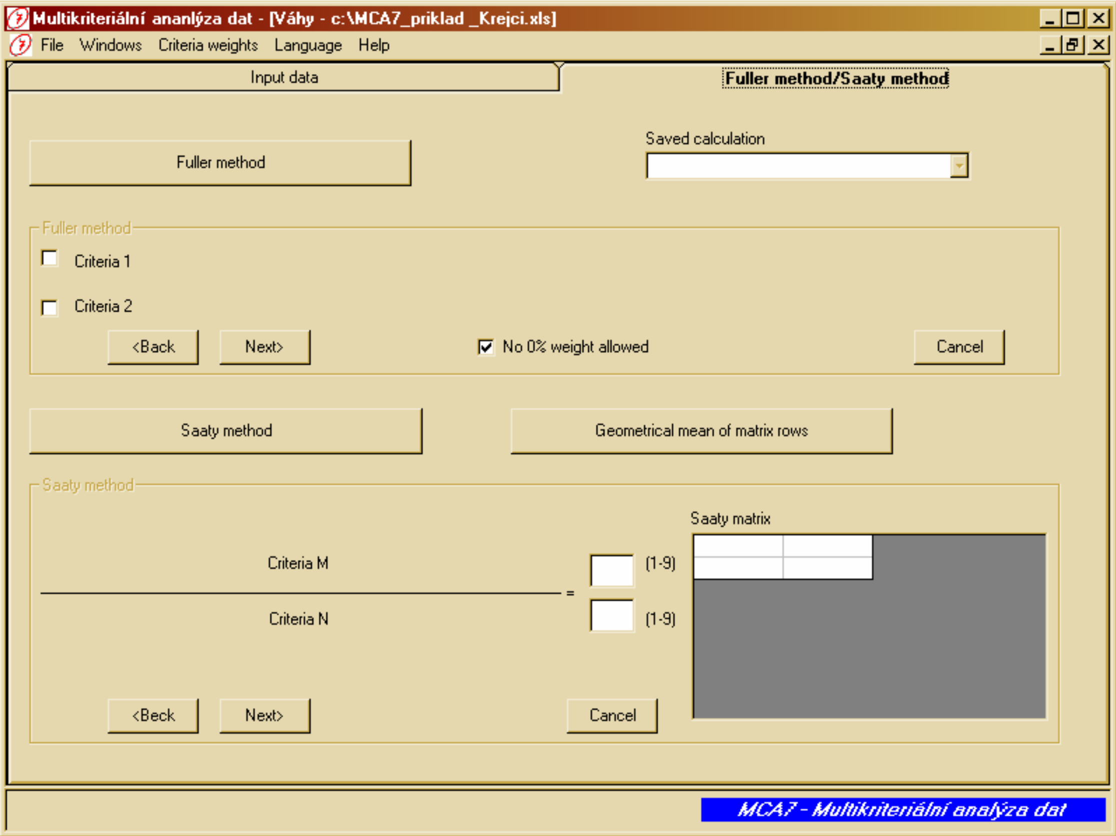


Fig. 3. Solution of criteria weights

Conclusions

The application of the systems of remote-controlled components causes acceleration in handling and thus shortening of duration of a fault in the network. This results in rising of probability of faultless service and thus the reliability of electrical energy supply. When deciding where to apply these components the multi-criteria analysis can be advantageously used. A weight is assigned to each criterion. It expresses the importance of particular criteria in relation to the others. The concordance-discordance analysis seems to be most objective from above mentioned methods from our practical experience. IPA, WSA and TOPSIS methods can be recommended to be used at the beginning of solving the investment designs. The CDA method is much more complicated and it is suitable for final decision-making for this reason with emphasis put on the objectivity of the final solution. In the course of calculation of criteria weights the greatest problem is the acquisition of input data. This data has subjective character caused by the reviewers; consequently the number of reviewers should be reasonable and the reviewers should know well the query.

Mentioned MCA methods will be furthermore analyzed in wider range for setting in distribution of electrical power for electrical network reliability increase.

Acknowledgment

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