Marek SZCZERBIŃSKI

Akademia Górniczo-Hutnicza, Katedra Elektroenergetyki

Lightning equivalent area of cuboidal structures assessed with electro-geometrical model

Streszczenie. (Piorunowa powierzchnia równoważna obiektów prostopadłościennych określana w oparciu o model elektro-geometryczny). *Piorunowa powierzchnia równoważna obiektu naziemnego jest definiowana jako powierzchnia gruntu, przejmująca taką samą liczbą wyładowań piorunowych, jaka trafia ten obiekt. Autor podjął się analizy powierzchni równoważnej z zastosowaniem tzw. „modelu elektro-geometrycznego”. W rezultacie otrzymał zbiór zależności m(h), charakteryzujących zależność zasięgu powierzchni równoważnej od wysokości obiektu dla różnych elementów budynku w formie prostopadłościanu (którym może być np. elektroenergetyczna stacja wnętrzowa), przy uwzględnieniu różnych wartości parametrów pioruna.*

Abstract*. Lightning equivalent area of a structure is defined as a ground area with the same annual frequency of strokes as to the structure. The author has undertaken research of the equivalent area analysis with use of the ‘electro-geometrical model’. As a result he got a set of m(h) dependencies (range of equivalent area vs. height of the structure) for various elements of the rectangular prism building (e.g. an electric power plant), taking into account different lightning parameters.*

Słowa kluczowe: piorunowa powierzchnia równoważna, model elektro-geometryczny.
Keywords: lightning equivalent area, electro-geometrical model.

Introduction

It is generally accepted that the number of lightning strokes to the structure results from the product of the annual density of lightning strokes N_g to the local ground surface and of the structure equivalent area A_e defined as a part of ground surface which intercepts the same annual number of strokes as the structure. The value of this area depends on the structure dimensions. Many different methods are proposed and some of them standardized to take into account those dependencies and influences, but the obtained results are very divergent, so next efforts to find acceptable solution seem to be acceptable [1, 2].

The product of the annual density of lightning strokes N_g and the structure equivalent area A_e determines the number of lightning strokes to the structure N.

(1) $$N = N_g A_e 10^{-6}$$

where: N_g [km $^{-2}$ year $^{-1}$] – ground flash density, A_e [m ^{2}] – lightning equivalent area of the structure.

The ground flash density N_g may result directly from lightning flash counter data for the region under consideration (if the results for long period of lightning activity are available). Usually it stems from isokeraunic level of the given region according to the relationship

(2) $$N_g = a T_d^b$$

where: T_d – annual number of thunderstorm days, a = 0.04, b =1.25 ((Some authors and standards take into account other values of the parameters a and b).

The simplest assessment of A_e is obtained for the structures with very small horizontal dimensions, as for instance: masts, towers or chimneys. In this case, the equivalent area may be expressed

(3) $$A_e = \pi m^2 h^2$$

where: h [m] – height of the structure, m – parameter of the value associated with h.

In order to simplify and unify the method of A_e assessment the dependence of m vs. h often is neglected and a constant value of m is used to be assumed. Naturally, the choice of such value is difficult and controversial but taking into account field and experimental data as well as different results of theoretical analyses it is possible in rough calculations to accept the value m = 3 as most adequate for the structures with a height till 60 m. This value determines the slope of the straight line going from the top of the structure to the ground surface. This line, at its movement around the mast, tower etc., will draw on the ground the circle of the radius 3 h.

In the case of structures with determined horizontal dimensions, the line drawing the border trace with constant slope touches successive points located around the roof on its top and ground surface. As it is easy to state for the structure of prism shape with dimensions W, L and H (Fig. 1.):

(4) $$A_e = LW + 6H(L+W) + 9\pi H^2$$

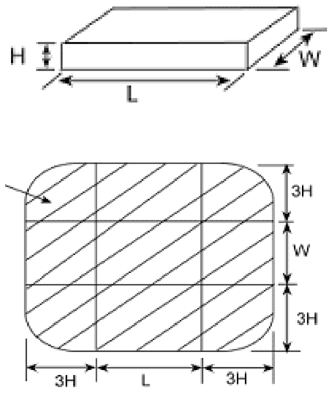

Fig. 1. Equivalent area A_e for the rectangular prism

The graphical method presented above enables in a relatively simple and universal way to assess the equivalent area A_e but the assumption that m = 3 is far-reaching simplification.

Model foundation

The model proposed by the author arises from the electro-geometrical theory of lightning attachment. The analytical expression that forms the basis for the electro-geometrical model of the lightning interception by a ground object has the commonly accepted form [3]

$$D = kI^c \tag{5}$$

where: D [m]– orientation distance, I [kA] – peak value of lightning current, k – parameter, c – parameter.

The above parameters, *k* and *c*, are estimated by different authors in the range: (3.3 < *k* < 15.3) and (0.65 < c < 0.85) [2].

From measurements taken by many researchers, sufficient data are available to show that the statistical distribution of the peak values of lightning current follows a logarithmic normal distribution. Thus, it is possible to approximate the probability density function of the orientation distances [4, 5]

$$f(D) = \frac{1}{cs\sqrt{2\pi}}\frac{1}{D}\exp\left[-\frac{1}{2}\left(\frac{1}{cs}\ln\frac{D}{D_{50\%}}\right)^2\right] \tag{6}$$

where: $D_{50\%}$ – median value of the orientation distance, s – standard deviation of the peak value of lightning current, c – parameter from the equation (5)

The values of standard deviation *s* from references are given in Table 1.

Table 1. Standard deviation *s* of the lightning current peak value

Type of lightning	Standard deviation *s* according Horváth [4]	Standard deviation *s* according to Flisowski [2]
Positive	s = 1.215	s = 0.97
Negative: I ≤ 20 kA I > 20 kA	 s = 1.325 s = 0.578	

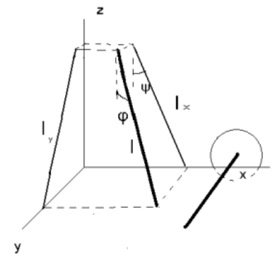

Fig. 2. Interpretation of angles *φ* and *ψ*; *l* - last segment of the step leader, *x* - horizontal line, l_y - component of *l* in the surface parallel to *x*, l_y - component of *l* in the surface perpendicular to *x*

The angle between the last step of the downward leader and the vertical *φ* is also a random variable but of disputable distribution. However, it is commonly accepted that the probability density function of its component perpendicular to a horizontal line (originally to an overhead high voltage line [6, 7], see Figure 2) has the form [8]:

$$f(\varphi) = \frac{2}{\pi}\cos^2\Psi \tag{7}$$

Quantitative results

The theoretical model developed by the author from (6) and (7) equations will be used in he following considerations and calculations. (Originally, the model has been developed for the mesh method effectiveness analyses [9, 10]).

The 'decision surface' [2] for elements of the prism rectangle is presented in Figures 3, 4 and 5. The annual frequency of lightning strokes attached by various elements of the structure may be computed with use the equations (8), (9), (10), (11), (12) and (13).

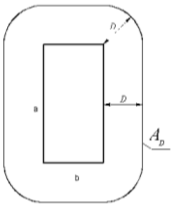

Fig. 3. Decision surface (top view)

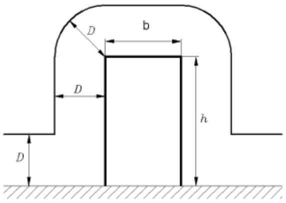

Fig. 4. Decision surface (side view) for *D* < *h*

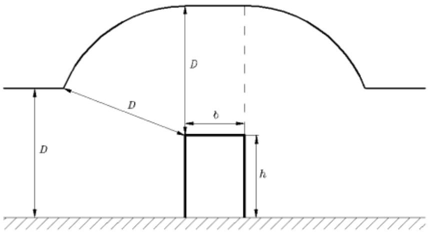

Fig. 5. Decision surface (side view) for D > *h*

Roof

$$N_0 = ab \tag{8}$$

where: N_0 [km $^{-2}$ year $^{-1}$] – annual frequency of lightning strokes attached by a roof surface (excluding the ridges), *a* [m] – width of the rectangle, *b* [m] – length of the rectangle

Horizontal ridge

(9)

$$N_1 = N_g \pi l \int_{D_{\min}}^{h} dD \int_0^{\pi/2} d\alpha \int_0^{\pi/2} D \frac{1}{cs\sqrt{2\pi}}\frac{1}{D}\exp\left[-\frac{1}{2}\left(\frac{1}{ps}\ln\frac{D}{D_{50\%}}\right)^2\right]\times$$

$$\frac{2}{\pi}\cos^2\psi\cos(\psi-\alpha)\sin\alpha\, d\psi +$$

$$N_g \pi l \int_{h}^{D_{\max}} dD \int_0^{\arccos(D-h/D)} d\alpha \int_0^{\pi/2} D \frac{1}{cs\sqrt{2\pi}}\frac{1}{D}\exp\left[-\frac{1}{2}\left(\frac{1}{ps}\ln\frac{D}{D_{50\%}}\right)^2\right]\times$$

$$\frac{2}{\pi}\cos^2\psi\cos(\psi-\alpha)\sin\alpha\, d\psi$$

where: N_1 [km $^{-2}$ year $^{-1}$] – annual frequency of lightning strokes attached by a section of l length of horizontal ridge of the roof (excluding he corners)

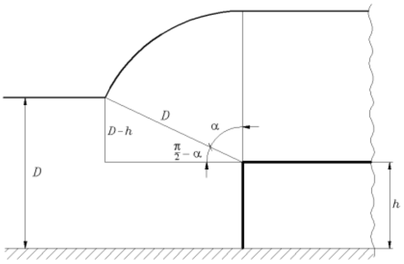

Fig. 6. Decision surface and the designations (side view) for *D* > *h*

Wall

(10)
$$N_2 = k_b N_g l \int_{D_{\min}}^{h} (h-D) g(D) dD =$$
$$0{,}25 N_g l \int_{D_{\min}}^{h} (h-D) \frac{1}{cs\sqrt{2\pi}} \frac{1}{D} \exp\left[-\frac{1}{2}\left(\frac{1}{cs}\ln\frac{D}{D_{50\%}}\right)^2\right] dD$$

where: N_2 [km $^{-2}$ year $^{-1}$] – annual frequency of lightning strokes attached by a wall of l length (excluding the ridges and corners), k_b – factor defined as follows (see Fig. 7):

(11)
$$k_b = \frac{\int_0^{\frac{\pi}{2}} \frac{2}{\pi}\cos^2\psi\cos(\psi-\frac{\pi}{2})d\psi}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2}{\pi}\cos^2\psi\cos\psi d\psi} = 0{,}25$$

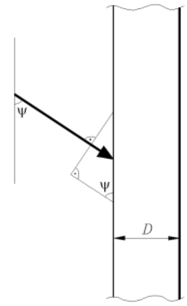

Fig. 7. Sketch for computation k_b

Corner

(12)

$$N_3 = \frac{1}{4} N_g 4\pi \int_{D_{\min}}^{h} dD \int_0^{\pi/2} d\alpha \int_0^{\pi/2} D^2 g(D) g(\psi)\cos(\psi-\alpha)\sin\alpha\, d\psi +$$
$$\frac{1}{4} N_g 4\pi \int_{h}^{D_{\max}} dD \int_0^{\arccos(d-h/D)} d\alpha \int_0^{\pi/2} D^2 g(D) g(\psi)\cos(\psi-\alpha)\sin\alpha\, d\psi =$$
$$\frac{1}{4} N_g 4\pi \int_{D_{\min}}^{h} dD \int_0^{\pi/2} d\alpha \int_0^{\pi/2} D^2 \frac{1}{cs\sqrt{2\pi}} \frac{1}{d} \exp\left[-\frac{1}{2}\left(\frac{1}{cs}\ln\frac{D}{D_{50\%}}\right)^2\right] \times$$
$$\frac{2}{\pi}\cos^2(\psi-\alpha)\sin\alpha\, d\psi +$$
$$\frac{1}{4} N_g 4\pi \int_{h}^{D_{\max}} dD \int_0^{\arccos(D-h/D)} d\alpha \int_0^{\pi/2} D^2 \frac{1}{cs\sqrt{2\pi}} \frac{1}{d} \exp\left[-\frac{1}{2}\left(\frac{1}{cs}\ln\frac{D}{D_{50\%}}\right)^2\right] \times$$
$$\frac{2}{\pi}\cos^2(\psi-\alpha)\sin\alpha\, d\psi$$

where: N_3 [km $^{-2}$ year $^{-1}$] – annual frequency of lightning strokes attached by a corner.

Vertical ridge

(13)
$$N_4 = k_b N_g \frac{2\pi}{4} \int_{D_{\min}}^{h} (h-D) D g(D) dD =$$
$$0{,}25 N_g \frac{2\pi}{4} \int_{D_{\min}}^{h} (h-D) \frac{1}{cs\sqrt{2\pi}} \frac{1}{D} \exp\left[-\frac{1}{2}\left(\frac{1}{cs}\ln\frac{D}{D_{50\%}}\right)^2\right] D dD$$

where: N_4 [km $^{-2}$ year $^{-1}$] - annual frequency of lightning strokes attached by a vertical ridge (excluding the corner)

The calculated annual frequency of lightning strokes attached by particular elements of the prism is influenced by the values of s, c, and $D_{50\%}$ parameters: see exemplary results for *h* = 50 m, presented in Tables 2, 3, 4, and 5.

Table 2. Annual frequency ***N₁*** of lightning strokes attached by a horizontal line for: *l* = 10 m, *h* = 50 m, N_g = 2.5 [km $^{-2}$ year $^{-1}$]. (The value N_g = 2.5 [km $^{-2}$ year $^{-1}$] is typical for Central Europe)

	$D_{50\%}$ = 50 m	$D_{50\%}$ = 100 m	$D_{50\%}$ = 150 m
s = 0.5 *c* = 0.65 (*sc* = 0.325)	$2.45\cdot10^{-3}$	$3.75\cdot10^{-3}$	$4.61\text{E}\cdot10^{-3}$
s = 1.0 *c* = 0.75 (*sc* = 0.75)	$2.55\cdot10^{-3}$	$3.86\cdot10^{-3}$	$4.73\cdot10^{-3}$
s = 1.5 *c* = 0.85 (*sc* = 1,275)	$2.77\cdot10^{-3}$	$2.78\cdot10^{-3}$	$4.30\cdot10^{-3}$

Table 3. Annual frequency ***N₂ of*** lightning strokes attached by a wall for: *l* = 10 m, *h* = 50 m, N_g = 2.5 [km $^{-2}$ year $^{-1}$]

	$D_{50\%}$= 50 m	$D_{50\%}$ = 100 m	$D_{50\%}$ = 150 m
s = 0.5 *c* = 0.65 (*sc* = 0.325)	$3.35\cdot10^{-5}$	$5.43\cdot10^{-7}$	$7.85\cdot10^{-9}$
s = 1.0 *c* = 0.75 (*sc* = 0.75)	$6.24\cdot10^{-5}$	$1.65\cdot10^{-5}$	$5.53\cdot10^{-6}$
s = 1.5 *c* = 0.85 (*sc* = 1.275)	$8.42\cdot10^{-5}$	$4.26\cdot10^{-5}$	$2.56\cdot10^{-5}$

Table 4. Annual frequency ***N₃*** of lightning strokes attached by a corner for: *h* = 50 m, N_g = 2.5 [km $^{-2}$ year $^{-1}$]

	$D_{50\%}$ = 50 m	$D_{50\%}$ = 100 m	$D_{50\%}$ = 150 m
s = 0.5 *c* = 0.65 (*sc* = 0.325)	$7.77\cdot10^{-3}$	$1.86\cdot10^{-2}$	$2.87\cdot10^{-2}$
s = 1.0 *c* = 0.75 (*sc* = 0.75)	$1.02\cdot10^{-2}$	$2.25\cdot10^{-2}$	$3.36\cdot10^{-2}$
s = 1.5 *c* = 0.85 (*sc* = 1.275)	$1.58\cdot10^{-2}$	$2.70\cdot10^{-2}$	$3.42\cdot10^{-2}$

Table 5. Annual frequency ***N₄*** of lightning strokes attached by a vertical ridge *h* = 50 m, N_g = 2.5 [km $^{-2}$ year $^{-1}$]

	$D_{50\%}$ = 50 *m*	$D_{50\%}$ = 100 *m*	$D_{50\%}$ = 150 *m*
s = 0.5 *c* = 0.65 (*sc* = 0.325)	$6.68\cdot10^{-3}$	$2.25\cdot10^{-3}$	$3.22\cdot10^{-4}$
s = 1.0 *c* = 0.75 (*sc* = 0.75)	$2.09\cdot10^{-3}$	$1.24\cdot10^{-3}$	$6.67\cdot10^{-4}$
s = 1.5 *c* = 0.85 (*sc* = 1.275)	$9.08\cdot10^{-4}$	$6.89\cdot10^{-4}$	$5.21\cdot10^{-4}$

Analysis of *m(h)* dependency

From the equations (8), (9), (10), (11), (12) and (13) one can conclude that the equivalent area A_e of the prism is given by the formula:

$$A_e = N_0 + 2\frac{a}{l}N_1 + 2\frac{b}{l}N_1 + 2\frac{a}{l}N_2 + 2\frac{b}{l}N_2 + 4N_3 + 4N_4 \quad (14)$$

Next, to compute m_w (i.e. the parameter m for the strips along the walls a and b) one can notice that

$$N_g m_w hl = N_1 + N_2 \quad (15)$$

thus

$$m_w = \frac{N_1 + N_2}{N_g hl} \quad (16)$$

Next, to compute m_c (i.e. the parameter m for the quarters of a circle around the corners) one can notice that

$$N_g \frac{\pi (m_c h)^2}{4} = N_3 + N_4 \quad (17)$$

Thus:

$$m_c = \frac{1}{h}\sqrt{\frac{4(N_3 + N_4}{\pi N_g}} \quad (18)$$

The results of the computations are presented in Table 6.

Table 6. Parameters m_w and m_c vs. the height h for s = 1.0, c = 0.75, and $D_{50\%}$ = 100 m

	m_w	m_c
h = 5 m	9.56	6.80
h = 10 m	6.86	4.84
h = 50 m	3.09	2.14
h = 100 m	2.04	1.43

Conclusions

1. The proposed model gives possibility to compute annual frequency of lightning strokes attached by various elements of the rectangular prism: horizontal and vertical ridges, walls corners. As a result, calculation of the equivalent area A_e and *m(h)* dependency is possible.
2. The parameters m_w and m_w vs. the height h are decreasing functions.
3. On the grounds of achieved results one can conclude that the value m = 3 for the structure height up to 60 m (see IEC Standards) seems to be underrated for the heights up to 50 m.

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Author: dr inż. Marek Szczerbiński, Akademia Górniczo-Hutnicza, Katedra Elektroenergetyki, al. Mickiewicza 30, PL 30-059 Kraków, Poland, E-mail: masz@agh.edu.pl;